69. Proposed by GEORGE LILLEY, Ph. D., LL. D., Professor of Mathematics, University of Oregon, Eugene, Oregon.

An elliptic fence encloses a field whose major and minor axes are $2a$ and $2b$, respectively. The ends of a rope, the length of which is equal to the length of the fence, are fastened outside the fence and at the extremities of the major axis. A horse is tethered by means of a ring which slides freely on the rope. Over how much ground can he feed? What is the length of the outside border? Find these values in square feet and feet, true to six decimal places, when the area of the field is one acre and $a=2b$.

Solution by MELLEN WOODMAN HASKELL, A. M., Ph. C., Associate Professor of Mathematics, University of California, Berkeley, Cal.

The outside border will evidently be a curve parallel to the given ellipse, so that the two curves will have common normals. Let ds denote the element of arc on the ellipse, dS the element of arc on the parallel curve lying between the same normals, $d\phi$ the angle between those normals, dA the element of area bounded by those normals, by ds and dS, and p the length of the rope, then

$$dS=ds+pd\phi, \text{ and } dA=pds+\tfrac{1}{2}p^2d\phi.$$

Integrating around the ellipse, since in this case the perimeter of the ellipse $\int ds$ is equal to p, and also the complete integral $\int d\phi$ is evidently 2π, we have

$$S=p(1+2\pi) \text{ and } A=p^2(1+\pi).$$

A simple calculation then gives

$p=806.693$ feet, perimeter of ellipse;
$S=5875.293$ feet, length of outside border;
$A=2695155$ square feet, included area.

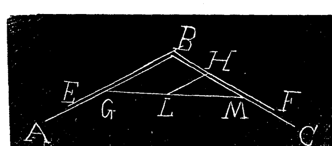
MECHANICS.

57. Proposed by J. C. NAGLE, M. A., M. C. E., Professor of Civil Engineering, Agricultural and Mechanical College of Texas, College Station, Texas.

Over the intersection of two inclined planes slides a cord of uniform mass throughout its length. Find the equation to the path described by its center of gravity.

Solution by the PROPOSER.

Let AB, BC be the inclined planes, EBF the position of the cord at any instant. Let the length of the cord be a; let BC be the x-axis, AB the y-axis. Let $EB=z$. Then the center of gravity of EB will be at G, the mid-point of EB, and of BF at M, the mid-point of BF. Let L be the center of gravity of whole cord. By moments about M,



$GM\times z=LM\times a$, or $GM/LM=a/z$.........(1).

But from similar triangles,

$$\frac{GM}{LM}=\frac{BM}{HM}=\frac{\frac{1}{2}(a-z)}{\frac{1}{2}(a-z)-x}=a/z.$$